Problem 1. **(1 point)** `Library/Rochester/setLinearAlgebra5LUfactorization/ur_la_5_2.pg`

Find the LU factorization of

$$A = \begin{bmatrix} -4 & 4 & 1 \\ 8 & -12 & 1 \end{bmatrix}.$$

That is, write $A = LU$ where L is a lower triangular matrix with ones on the diagonal, and U is an upper triangular matrix.

$$A = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix} \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix}$$

Problem 2. **(1 point)** `Library/Rochester/setLinearAlgebra5LUfactorization/ur_la_5_3.pg`

Find the LU factorization of

$$A = \begin{bmatrix} -1 & 1 \\ -3 & 6 \\ -2 & -4 \end{bmatrix}.$$

That is, write $A = LU$ where L is a lower triangular matrix with ones on the diagonal, and U is an upper triangular matrix.

$$A = \begin{bmatrix} \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \\ \_\_ & \_\_ & \_\_ \end{bmatrix} \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$$

Problem 3. **(1 point)** `Library/Rochester/setLinearAlgebra5LUfactorization/ur_la_5_7.pg`

Find the LU factorization of

$$A = \begin{bmatrix} 1 & -5 \\ 1 & -9 \end{bmatrix}$$

and use it to solve the system

$$\begin{bmatrix} 1 & -5 \\ 1 & -9 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} 22 \\ 38 \end{bmatrix}.$$

$$A = LU = \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix} \begin{bmatrix} \_\_ & \_\_ \\ \_\_ & \_\_ \end{bmatrix}$$

$$\vec{x} = \begin{bmatrix} \_\_ \\ \_\_ \end{bmatrix}$$

Problem 4. **(1 point)** `METUNCC/Linear_Algebra/LU_Divide-3x3.pg`

In this problem you will use LU decomposition to divide

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} -2 & -3 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ 2 \end{bmatrix}$$

Step 1. Divide by L.

Use forward substitution to solve

$$\begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ 1 & 3 & 1 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} -2 \\ 2 \\ 2 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{bmatrix}$$

Step 2. Divide by U.

Use back-substitution to solve

$$\begin{bmatrix} -2 & -3 & 3 \\ 0 & 1 & -1 \\ 0 & 0 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$\begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} \_\_\_ \\ \_\_\_ \\ \_\_\_ \end{bmatrix}$$

Hint: All answers should simplify to be integers.